Appendix – definitions and explanations (only online as supplementary material)

Stress

Stress $\sigma$ is equal to the applied force per unit area. Pressure has the same units, and indeed is a type of stress, but stress more generally can be a vector, that is, it is applied in a specified direction, whereas pressure acts equally in all directions.

Strain

Strain $\varepsilon$ is the consequence of applying stress to an elastic medium. It is equal to the amount of deformation experienced by an element of the medium compared to its original size and shape. In elastography it is often calculated as the spatial gradient of displacement. It is a dimensionless quantity, i.e. it has no units. Volumetric strain (or dilatation) results from a change in pressure. Shear strain results from a directional stress.

Strain rate

Strain rate $\frac{d\varepsilon}{dt}$ is the rate of change of strain. In elastography it often results from calculating the spatial gradient of tissue velocity, obtained using tissue Doppler methods.

Bulk modulus

The bulk elastic modulus $K$ is the tissue property that determines the amount of volumetric strain (relative change in density) produced by a given pressure.

Shear modulus

The shear elastic modulus $G$ is the tissue property that determines the amount of shear strain (relative change in shape) produced by a given directionally applied stress.

Young’s modulus

The usual form of Hooke’s law describes longitudinal stress $\sigma$ (compressional or tensional) as generating a proportionate strain $\varepsilon$ that is a relative change in length, where the Young’s modulus $E$ is the constant of proportionality:

$$E = \frac{\sigma}{\varepsilon}$$

Poisson’s ratio $\nu$ is equal to the lateral strain (expansion) divided by the axial strain (shortening) of an object under axial compression. The relationship between the Young’s modulus and the shear modulus can be expressed as:

$$G = \frac{E}{2(1+\nu)}$$

Incompressible materials have a Poisson’s ratio of 0.5. Soft tissues are often regarded as nearly incompressible, which will be true so long as the strain occurs sufficiently quickly that tissue fluids do not have time to move. For many practical purposes, therefore, the relation between Young’s modulus and shear modulus is:

$$E \approx 3G$$

Stiffness

Stiffness is often, but incorrectly, used as a synonym for elastic modulus. It is the force with which an object resists surface displacement, measured as force divided by distance. It increases with the Young’s modulus of the material but also depends on the shape of the object and whether it is free to strain at its boundaries.

Speeds of sound waves and shear-waves

Sound speed is given by

$$c_s = \sqrt{\frac{K}{\rho}}$$

For soft tissues, which are not very rigid, G is about a million times smaller than K. Therefore, the speed of an ultrasound wave in tissue is almost completely determined by the compressional elastic modulus and density, i.e. it is usually assumed that for soft tissues and liquids

$$c_s = \sqrt{\frac{G}{\rho}}$$

Given the ratio of $G$ to $K$ mentioned above, the square root relationship between elastic modulus and wave speed, and similar tissue density, $c_s$ for soft tissues is typically about 1000 times smaller than $c_s$.

Material nonlinearity

A material that obeys Hooke’s law, where stress is proportional to strain, is said to be linear. Biological tissues in general exhibit nonlinear behaviour, as in Fig. 18. If the stress is applied in two intervals, each with a different amount of prestress of the tissue/material, the resulting strain is different, the strain being less for a given interval in stress the greater the prestress. In other words, tissue stiffens the more it is stressed. For displacement imaging or strain imaging, the greater the prestress the smaller will be the achieved displacement or strain for a given additional applied force. For shear-wave methods, wave speed increases with the prestress.

Material viscoelasticity

Materials that obey Hooke’s law are also said to be purely elastic. Biological tissue in general, however, exhibits viscoelastic behaviour; at low strain rates, molecular structural “flow” can occur, but this does not have time to happen at high strain rates. Therefore materials stiffen the faster they are strained. Thus, for shear-wave methods, the use of higher vibration frequencies will result in higher values of wave speed [69]. Viscous effects are also responsible for loss of energy from the shear-waves as they travel, and shear-wave attenuation increases with shear-wave frequency. At very low (quasi-static) rates of strain, i.e. for strain imaging, the viscoelastic nature of tissue may manifest in other ways. Sustaining a constant applied force causes the material to “creep”, i.e. the strain continues to slowly increase in magnitude [70]. If this happens during the compression phase of a slow palpation cycle (Fig. 5), the strain remains elevated during the release phase, resulting in the phenomenon of hysteresis where the stress-strain curve in Fig. 18 follows a different path for compression and release. The material may also “remember” being strained if the next strain cycle occurs before the tissue has re-
turned to its original state [73]. Finally, in the behaviour known as stress relaxation, if the strain were to be held constant, the applied force needed to create this strain would decrease with time.

Material poroelasticity
Sometimes confused with viscoelasticity is the phenomenon of poroelasticity, a clinical example of which is pitting oedema. If strain occurs sufficiently slowly that tissue fluids have time to move, tissue may become compressible because its volume can be reduced as fluid is forced out. This is a mechanism for causing the above phenomena of creep and stress relaxation, but Poisson’s ratio also reduces and becomes time dependent [74]. In strain imaging, if the transducer-induced tissue surface displacement is held constant, poroelastic effects cause the strain image to change with time [75].

Material anisotropy
Many tissues are structurally anisotropic, such that stiffness and shear-wave speed depend on the measurement direction [73, 76, 77].

Material discontinuities
Slippery boundaries between tissues may generate high strain at the boundary and reduce or prevent both shear-wave and strain penetration across the boundary [78, 79].

Shear-wave scattering, reflection and refraction
Variations in either tissue density or shear-wave speed will cause scattering, reflection and refraction of shear-waves, which may generate errors in shear-wave speed estimation. Scattering results in shear-wave attenuation and a shear-wave speed that depends on vibration frequency, but the contribution to these phenomena from scattering, relative to that from viscoelastic effects, is not yet well understood.

Guided shear-wave propagation
At tissue surfaces and interfaces a mechanical disturbance may propagate as a guided wave that follows the structure. Various types of such wave, which travel with a speed different to that of a pure shear-wave, are possible. For example, at or near a surface or interface, Rayleigh waves (where the tissue displacement pattern follows an elliptical path that orbits the equilibrium position) will travel at a speed that is slightly slower than the pure transverse vibrations shown in Fig. 3. Within tissue layers (such as the myocardium, artery walls, or in the skin) Lamb waves (displacement perpendicular to the surfaces) or Love waves (displacement parallel to the surfaces but still perpendicular to the propagation direction) may be possible, where the wave speed depends not only on $G$ but on the ratio of the wavelength and the thickness of the layer. Such phenomena may be the source of artefact in SWE when the shear modulus is being measured, but correction methods are under development to enable reliable tissue viscoelastic property measurement (e.g. [80–82]).

References
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